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Electromagnetic interactions of cosmic ray muons in iron II. Momentum dependence of the interaction probabilities

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Abstract. The Durham spectrograph MARS has been used to study the electromagnetic interactions of cosmic ray muons in iron in the energy range 6–200 GeV. The interaction probabilities for producing single electrons and electron bursts of various sizes from the knock-on process, direct electron pair production and bremsstrahlung are investigated as a function of muon energy. Reasonable agreement between the measured probabilities and those predicted has been found and it is concluded that significant deviations from accepted theory are not likely.

1. Introduction

An experiment has been carried out with the Durham spectrograph MARS to investigate the electromagnetic interactions of cosmic ray muons in iron with particular reference to the search for any asymmetry in the interaction probabilities dependent on the charge of the muon and a study of the energy dependence of the overall probabilities. This paper is concerned with a determination of the magnitude of the interaction probabilities; the previous paper (Grupen *et al* 1972, to be referred to as I) related to the asymmetry search.

The electromagnetic interactions contributing to the single electron and electron burst events are the muon–electron knock-on process, direct electron pair production, bremsstrahlung and to a very small extent nuclear interaction through the virtual photon flux associated with the muon. The study of the electromagnetic interactions of muons is of interest, not only for the purpose of testing the theories concerning the various processes but also in view of the fact that the overall energy loss of the muon and hence the important range–energy relation is derived from a knowledge of these processes. Precise information about the range–energy relation is required for the interpretation of a number of cosmic ray experiments, particularly those concerned with underground measurements. Finally, it is necessary to measure interaction probabilities in iron so that corrections may be applied to magnetic spectrograph data in experiments to measure the momentum spectrum of muons.

A number of experiments with cosmic ray muons have been performed in the past to measure the knock-on and direct pair production processes (see the recent paper by Allkofer *et al* 1971 for a summary).

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For the knock-on process, experiments by Deery and Neddermeyer (1961), Chaudhuri and Goswami (1970) and Allkofer *et al* (1971) have suggested a small deviation from the theory of Bhabha (1938) for energy transfers around 1 GeV while other experiments, by McDiarmid and Wilson (1962) and Kirk and Neddermeyer (1968), have found good agreement. As for direct pair production, the experiments (see Allkofer *et al* 1971 for a summary) have indicated conflicting results, although the measurements of highest statistical precision, by Kearney and Hazen (1965) and Chaudhuri and Sinha (1964), give results consistent with expectation. Furthermore, the results of Kelly *et al* (1968), in a study similar to the present one give no indication of anomalies.

It is difficult with cosmic ray muons to study the bremsstrahlung process with accurate statistics in view of the fact that, compared with other processes, the interaction cross section is relatively small except for very large fractional energy transfers. Some work has been carried out at very high energies by Matano *et al* (1968) and Alexander *et al* (1969), but this is somewhat indirect: showers of electrons are observed in the atmosphere at large zenith angles and they may be due to bremsstrahlung or to nuclear interactions. Whichever is the source, Matano *et al* consider that their number is in excess of expectation.

In the present work, interactions produced in the magnet iron of the MARS instrument have been studied and some measure of distinction between the knock-on and pair production processes has been achieved by studying the energy dependence of the probabilities.

2. Experimental arrangement

A detailed description of the characteristics and the operation of the cosmic ray spectrograph MARS has been given by Ayre *et al* (1972) and a summary of the present experimental arrangements and the method of data reduction have been given in I.

3. Theory and predictions

The basic information required for the analysis of the experimental data includes the electromagnetic shower development curves for iron, and the interaction cross sections for the various processes as a function of muon energy and energy transfer. The energy spectrum of muons at each interaction level is also needed for part of the analysis. The theoretical cross section for knock-on production has been given by Bhabha (1938). The differential cross section is weakly dependent on the muon energy and varies inversely as the square of the energy transfer. This interaction gives rise to the majority of the observed events in the first run of the present experiment. The interaction cross sections for the direct pair production and bremsstrahlung processes which we have used in our analysis are those given by Murota *et al* (1956) with the indefinite constant $\alpha = 2$, and Christy and Kusaka (1941) respectively. Of the three kinds of electromagnetic interactions mentioned above, the cross section for direct pair production increases more rapidly with muon energy than the other two, but it falls off very rapidly with energy transfer. The cross section for bremsstrahlung, however, is relatively small and decreases rather slowly with increasing energy transfer.

The differential cross sections are given in figure 1 as a function of muon energy for two representative energy transfers: 1 and 10 GeV. From this figure it follows that pair

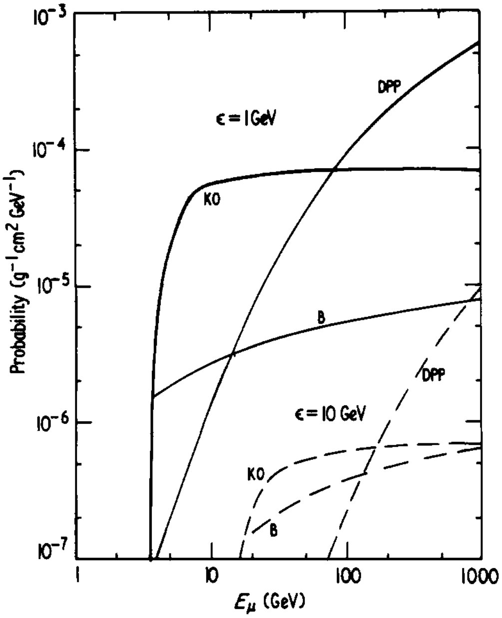

Figure 1. Interaction probabilities for the three processes, knock-on production (KO), direct pair production (DPP) and bremsstrahlung (B) as a function of muon energy for two energy transfers.

production predominates at an energy transfer, $\epsilon = 1$ GeV for muons of energy $E_\mu > 70$ GeV and other plots, not shown, indicate predominance for $E_\mu > 30$ GeV at an energy transfer of 0·3 GeV. Bremsstrahlung is the main process for $\epsilon = 1$ GeV and $E_\mu \lesssim 4$ GeV, $\epsilon = 10$ GeV and $E_\mu \lesssim 17$ GeV. For $E_\mu = 100$ GeV, bremsstrahlung predominates for $\epsilon \gtrsim 20$ GeV.

The contribution from the nuclear interactions of the muons is insignificant in the present experiment and has not been included.

In order to calculate the expected burst spectra at the different measuring levels, the energy spectra of the muons incident upon the interaction region (a small thickness of iron above the level) must be known. The muon energy spectra at the two measuring levels have been calculated using the vertical energy spectrum previously measured at Durham (Ayre *et al* 1971), the known acceptance of the apparatus and the energy loss in each iron block. (The spectrograph determines momentum, not energy, but the distinction is ignored.) The spectra are shown in figure 2. It is expected that the burst spectra measured at the levels 1 and 3 will be somewhat different on account of these differing muon energy spectra.

Electromagnetic shower development curves which show the total number of electrons as a function of depth for iron have been reported by Ivanenko and Samosudov (1967). With this information and the interaction cross sections, the burst spectra for various individual muon energies have been calculated. By folding the muon energy spectra into these burst spectra, the expected integral burst spectra for measuring levels 1 and 3 have been found. They are shown in figures 3 and 4 respectively. In these calculations a threshold energy of 2 MeV for the electrons in the shower has been used as estimates show that this is the minimum energy for the electrons to be detected in the

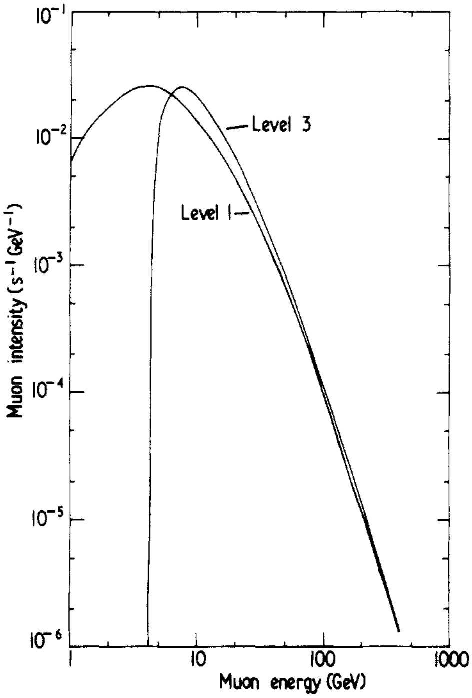

Figure 2. Energy spectrum of muons at the two levels in MARS where interactions are studied.

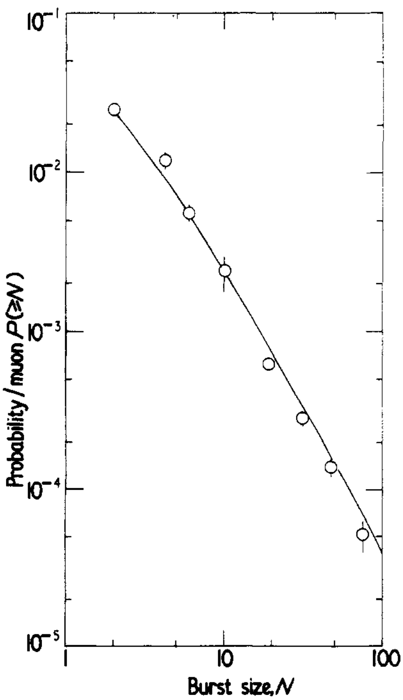

Figure 3. Integral burst size distribution for level 1. The full line is the theoretical prediction (not normalized).

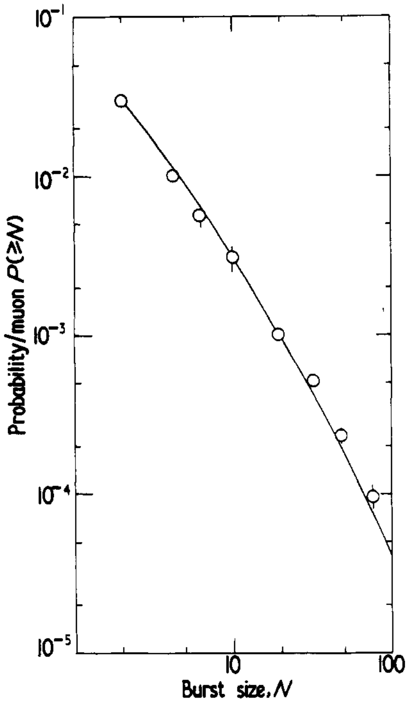

Figure 4. Integral burst size distribution for level 3. The full line is the theoretical prediction (not normalized).

flash-tube trays. Transition effects associated with the non-iron absorbers (scintillator and momentum selector trays) between the magnets and the 'measuring trays' of flash tubes have been neglected in this analysis. It is not expected that they will be large.

The results of calculations of burst frequencies for different unique muon energies are given in figure 5. Fluctuations in the distribution of electrons in showers have some effect on the absolute values of the burst frequencies. As most observed bursts are produced by showers near their maximum development, it is expected that the fluctuations approximate to the Poisson distribution and this has been used. The effect of fluctuations is to increase the burst frequencies by an amount which is greater for lower muon energies where the slope of the differential cross section as a function of energy transfer is steeper. The present results agree with those obtained by Crawford and Messel (1970) for small shower sizes where comparison can be made.

Approximate calculations have been made for the probabilities of observing single secondary electrons in which the contributions from the knock-on and direct pair production processes have been included. The resulting probability against muon energy is given in figure 6. The reason for the lack of interest in these low energy events is that these probabilities have been the subject of considerable study over the years (eg by Lloyd and Wolfendale 1959) and there is no evidence for any untoward behaviour.

4. Analysis of the experimental data

Two sets of data were obtained. One with discriminator levels set to record at least one

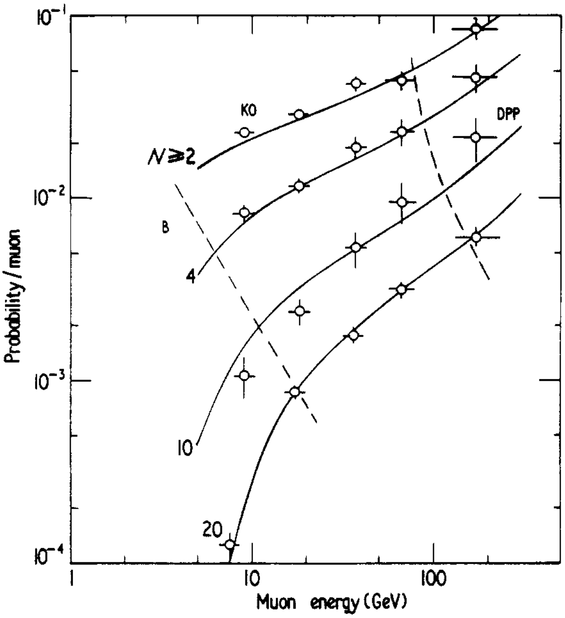

Figure 5. Integral burst size probabilities as a function of muon energy. Results for $N \geqslant 2$, 4 and 10 come from run 1 and those for $N \geqslant 20$ are derived from run 2. The full lines represent theoretical expectation. The regions of predominance are shown. The horizontal errors represent the standard deviation on the energy estimates (they are greater than normal for MARS, at high energies, because the detected burst commonly obscures the muon track in one measuring tray).

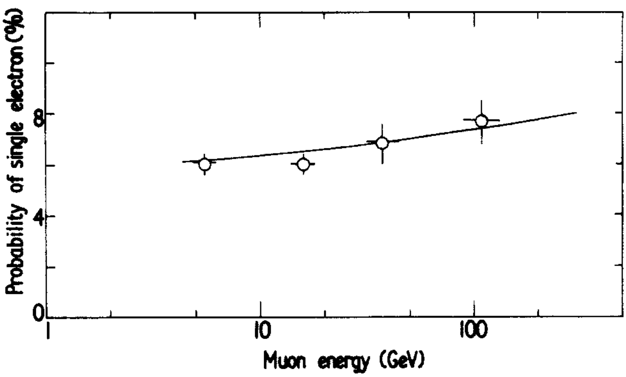

Figure 6. Probability of observing a single electron as a function of muon energy. The full line represents an approximate theoretical estimate.

particle and a second with the levels set at six equivalent particles. The first set comprising 2363 single electron events and 1117 burst events was obtained with a total effective running time of 8 hours during which time $9{\cdot}36 \times 10^3$ muons traversed the apparatus. The second set of data (on the 6 particle setting) with 636 single electron events and 2097 burst events was based on a total running time of 350 hours. The number of muon traversals was $3{\cdot}95 \times 10^5$.

The number of electrons in each event, that is the burst size, has been determined by examining the pattern of flashes in the appropriate flash-tube tray. Rogers (1965) introduced a simple statistical method of allowing for multiple traversals of flash-tubes in showers of constant density and this has been modified in the present work and applied to the case of bursts with their rapidly falling lateral density distribution. The result is that the size of an individual burst (of say 10 particles) can be estimated from the pattern of flash-tubes discharged to an accuracy of about 25%.

It is not possible in the present experiment to distinguish between the various kinds of electromagnetic interactions for individual events. However, since the majority of bursts detected are near the maximum of their development and a rough correlation exists between N and ϵ ($\epsilon(\mathrm{GeV}) \simeq \frac{1}{4}N$); then, for a group of particles in a cell of E and N, the cross section curves (eg figure 1) indicate the relative contributions from the various processes. These arguments have been used to give the approximate regions of predominance for the three interactions and the results are given in figure 5.

5. Comparison of experiment and theory

The measured burst spectra at levels 1 and 3 are shown in figures 3 and 4 respectively, together with the theoretical predictions. Below $N = 15$ the data come entirely from the first run but for burst sizes greater than 15 both runs are used as the detection of these showers is not affected by the discriminator setting. The considerably longer running time for the second run means that the bulk of the large burst size data came from this experiment. It can be seen that the spectrum for level 3 is higher than that for level 1, as expected, and they both agree reasonably well with predictions.

The interaction probabilities for bursts and for single electrons as a function of muon energy are shown in figures 5 and 6 respectively. The overall data were used for burst size $N > 20$ and data from the first run only were used for $N > 2$, 4 and 10.

It will be noted that integral plots are given (figures 3, 4 and 5); this is permissible in the present case where the spectra are falling so steeply that the degree of dependence of one point on those below it is very small indeed. Furthermore, the points below $N = 15$ are essentially independent of those above because they are drawn from the separate experimental runs.

For single electron production only data recorded on levels 2 and 4 were used, the flash-tube trays at these levels being just underneath the magnet blocks.

In both figures 5 and 6 the overall agreement between experiment and theory can be regarded as satisfactory.

Turning to the specific processes it can be seen that for the region occupied by each process the experimental points are distributed around the theoretical curves in a manner fully consistent with expectation. It cannot be stated that definite inconsistencies have been found for any of the processes although it should be pointed out that the bremsstrahlung region has not been covered to any great extent.

6. Comparison with previous experiments and conclusions

As was remarked in the Introduction although some workers have found agreement, others appear to have found discrepancies between the interaction probabilities and expectation. Insofar as we have measured essentially the total interaction probabilities, comparison will be made with those similar experiments and not with those in which positive distinction between knock-on electrons, direct electron pairs etc was possible.

Previous experiments have been carried out at a variety of zenith angles (θ) at sea level (SL) and underground (UG) and different target materials have been used. Those agreeing with expectation, and thus with the present data, include measurements by the following (see Hamdan 1972 for a more complete summary):

Ashton *et al* (1968), θ = 50°–90°, SL, iron target
Barton *et al* (1966), θ = 0°, UG, lead
Chaudhuri and Sinha (1964), θ = 0°, UG, iron and lead
Chin *et al* (1969), θ = 0°, UG, iron and lead
McDiarmid and Wilson (1962), θ = 0°, SL, iron and lead
Misaki *et al* (1969), θ = 90°, UG, lead.

In a similar number of experiments discrepancies are seen and these measurements are summarized in figure 7. In figure 7 the ratio R of observed frequency to expected frequency is plotted against energy transfer. Although such a plot is of rather limited value in view of the differing muon energy spectra and target elements used, it does set the scale of the deviations.

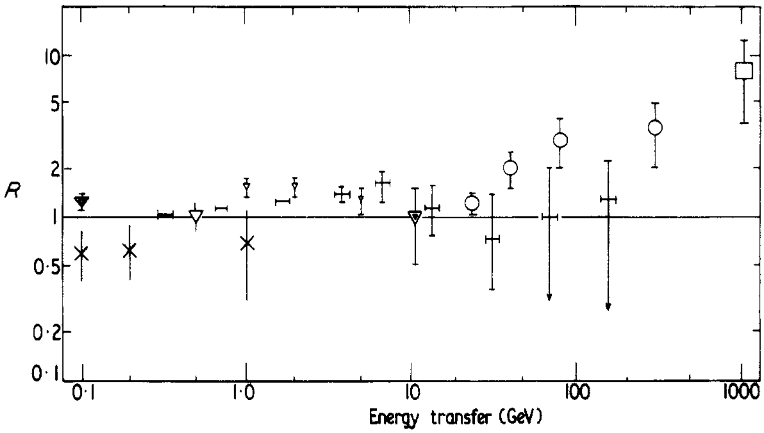

Figure 7. Previous results on the summed interaction probabilities which do not agree with expectation. The ordinate is the ratio R of the observed frequency to that expected.

Key	Experiment	Location	Target
○	Alexander *et al* (1969)	SL $\bar{\theta}\sim 73°$	Air
□	Matano *et al* (1968)	SL $\theta > 70°$	Air
⊢	Allkofer *et al* (1971)	SL $\bar{\theta}\sim 83°$	Iron
×	Gaebler *et al* (1961)	UG $\bar{\theta}\sim 0°$	Lead
▽	Kearney *et al* (1965)	UG 0° and 60°	Lead

(θ is the zenith angle, SL denotes 'sea level' and UG denotes 'underground').

A comment that can be made about all the experiments, and our own is no exception, is that there is difficulty in determining the energy transfer in the interactions and a small error in energy transfer produces a large uncertainty in absolute probability because of the rapidly falling spectrum of the bursts. It is likely that a number of the discrepancies are due to these errors. Indeed, it has been suggested by Kiraly *et al* (1971) that the high points of Alexander *et al* (1969) and part, at least, of the excess observed by Matano *et al* (1968)—this excess is the largest shown in figure 7—are due to errors in burst size determination and they give a detailed analysis in terms of normal bremsstrahlung behaviour.

Our conclusion is that the present work indicates quite good agreement between experiment and theory, the dominant processes being knock-on production and direct pair production, and we see no firm evidence for significant discrepancies from any of the other experiments carried out so far.

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